ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 9, Issue 1, January 2020

A Theoretical Moding of MHD Flow for Steady Flow of Blood through a Catheterized Arterial Stenosis with Velocity Slip

Dr. Mokshed Ali

Asst. Professor, Department of Mathematics, Barkhetri College, Assam, India

ABSTRACT: This article is concerned with the analysis of blood flow through arterial catheterization (widening of arteries by a balloon in the presence of mild stenosis), when blood is represented by MHD Newtonian fluid. A uniform magnetic field is applied perpendicular to the porous artery. A mathematical model is developed and analyzed by using appropriate mathematical techniques. Analytical expressions are obtained for axial velocity, flow rate, wall shear stress, and apparent viscosity and their variations with different flow parameters are illustrated in figures. In this confined annular region, the behavior of these flow variables has been explored. Its worth noting that, due to the introduction of an axial slip, velocity and flow rate will increase on one hand while wall shear stress and apparent viscosity will decrease on the other.The effects of stenosis and tapering are discussed. The study of the above model is very significant as it has direct applications in the treatment of cardiovascular diseases.

KEYWORDS: Laminar flow, catheterized tapered artery, viscosity.

I. INTRODUCTION

The study of blood flow through tapered tubes is important not only for an understanding of the flow behavior of the marvelous body fluid in arteries, but also for the design of prosthetic blood vessels (How and black,1987). It is reported that under certain diseased conditions, an abnormal and unfamiliar growth is developed at the lumen of an artery. This is usually termed as stenosis which is a kind of CVS disease, called atherosclerosis. This unwanted growth and its presence at one or more locations in blood vessels, causes serious circularatory disorders due to the reduction of blood supply to different organs and tissues or occlusion of an artery (Young 1970, Caro 1973). As stenosis develops through stages mild, moderate and severe, there arise complications in the body due to artery block partially or wholly or completely cut off of blood supply or fully scaled off of an artery. This results in different cvs diseases like, myocardial infarction, heart block, stroke, coronary thrombosis etc. (Boyd 1963, Guyton 1970, Puniyani and Nimi 1998, Dintenfass 1981, Biswas 2000). Apart from this, sometimes for medical point of view and, for clinical and subclinical purposes, artificial catheters are inserted in uniform and stenosed arteries. Pressure-flow relationship changes for blood flow through a stenosed artery and it will aggravate with the insertion of a catheter in the stenotic region as it will further increases the impedance or resistance to flow, alter the pressure distribution (Biswas and Bhattacherjee 2003, Biswas and Chakraborty 2010). It seems important to examine blood flow through stenosed artery and catheterized constricted artery. It is indicated that hydrodynamic factors could play an important role in the formation, development and progression of an arterial stenosis (Young and Tsai, 1979; caro,1971).

Sometimes for many useful medical and clinical purposes, artificial catheters are inserted in stenosed arteries. The pressure-flow relationship alters considerably due to the presence of a flexible catheter in the constricted region of an artery as this will lead to further enhancement of the impedance or frictional resistance to flow as well as will alter the pressure distribution (Young and Tsai,1979); Biswas and Nath,1999). Study of blood flow in the annular region of a mammalian artery has already drawn a good deal of interest and attention, among the researchers. In view of its potential physiological significance and a lot of clinical applications. It may be worth mentioning that in certain flow situations, physiological fluid blood may behave as a Newtonian fluid (Schlichting, 1968). The laminar flow of a

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 9, Issue 1, January 2020

Newtonian fluid through an annulus has been dealt in (Yuan, 1969). The annular blood flow in a catheterized stenosed uniform artery has been investigated in (Young and Tsai, 1979; Biswas and Nath,1999; Jayaraman and Tewari, 1995; Biswas and Bhattacherjee,2003). Recently blood flow in tapered vessels with the presence of a stenosis is modeled (Liu et al., 2004).

In recent years, theoretical, numerical and experimental investigations (Biswas,2000; Fung,1981; Forrester and Young,1970; Liu et al.,2004;How and Black,1987) have been performed to analyze blood flow in arteries. The complex geometry of arteries (viz. bending, bifurcating, branching, tapering, discrete etc.) is also an important factor which obviously affects the local hemodynamic (Guyton, 1970; Puniyani and Niimi,1998). The relationship between flow in arteries, particularly the wall shear stress, and the sites where atherosclerosis develops has motivated much of the research on arterial flow in recent decades. It is now well accepted that the sites where shear stresses are low or change rapidly in time or space, that are most vulnerable. These conditions are likely to prevail at places where the vessel is curved, bifurcated, has a junction, a side branch or other sudden change in flow geometry 9Berger and Jou,2000). There is no doubt that tapering in arteries, is a significant aspect of mammalian arterial system and the formation of stenosis along the tapered wall, may alter the flow situation to a great extent. In view of the above considerations, we are interested to study the steady annular flow of blood through a catheterized tapered vessel with stenosis. In this case, blood is taken as a Newtonian fluid and an axial slip velocity is introduced at the stenotic wall of the tapered artery.

II. MATHEMATICAL FORMULATION

Flow Geometry and Co-ordinate system:
Figure 1 depicts the flow geometry of stenosis inserted Catheterized artery for the steady flow of an incompressible Newtonian fluid in an annular region between a constricted tube of undisturbed radius R_0 and a co-axial uniform circular pipe of radius kR_0 (k<<1).

Let (r, θ, x) be the cylindrical polar coordinate system with $r = 0$ as the axis of symmetry of the tube. Consider the flow of an incompressible magnetohydrodynamic (MHD) Newtonian fluid of constant viscosity μ and density ρ through coaxial tubes such that the outer tube with an axially non-symmetric but radially symmetric mild stenosis having length L and the inner tube have a balloon (angioplasty) on its wall and assume that the balloon model is axi-symmetric in nature and u and w are the velocity component in r and z directions, respectively. The stenosed wall and the Catheter model are defined by the functions $h(z)$ and $R(z)$ respectively. The geometry of the stenosis is defined as Mekheimer and El Kot [23]

$$h(z) = d(z)[1 - 5(b^{n-1}(z-a) - (z-a)^n)],\ a \le z \le a + b$$

$$= (z), \text{ otherwise} \qquad (1)$$

with $(x) = d_0 + sz$, where (z) is the radius of the tapered arterial segment in the stenotic region, d_0 is the radius of the non-tapered artery in the non-stenotic region, is the tapering parameter, b is the length of the stenosis, (≥ 2) is a parameter determining the shape of the constriction profile and referred to as the shape parameter (the symmetric stenosis occurs for $n = 2$) and a indicates its location.

The geometry of the catheter is defined as Mekheimer and El Kot [14]

$$(z) = d_0[k + \eth_2 \exp(-\pi^2(z - z_d - 0.5)^2],\ a \le z \le a + b$$

$$= kd_0, \quad \text{otherwise} \qquad (2)$$

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 9, Issue 1, January 2020

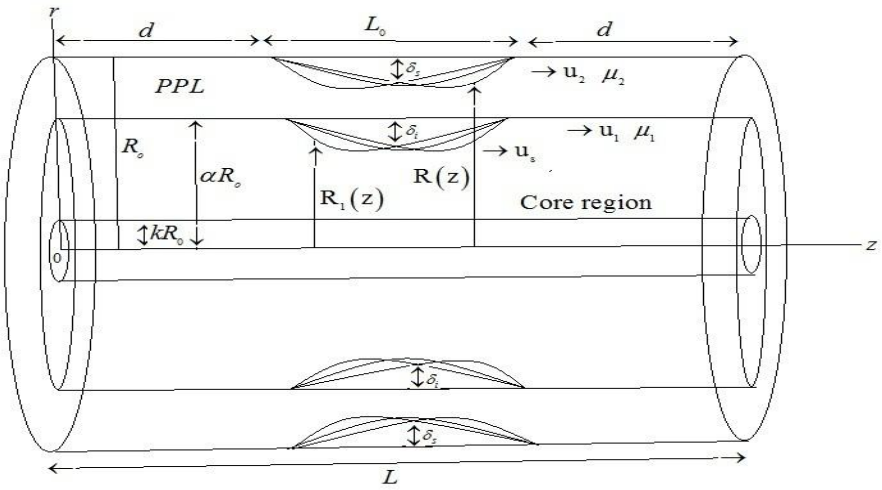

Fig 1:Schematic diagram for Annular Flow of Two-Layered Newtonian Fluid within catheterized Asymmetric Stenosed Artery

1. Governing Equations and boundary conditions:

Let us consider a steady, Laminar flow of blood through an axially non symmetric but radially symmetric stenosed artery - a circular tube with a catheter of radius kR_0 (k<<1) co-axial to it and one-dimensional flow obeying the constitutive equation for a Newtonian fluid. Fluid velocity vector has the form $\vec{V} = (0, 0, u(r))$ in cylindrical polar system (r, θ, z) representing the radial, circumferential and axial coordinate respectively. The equations of motion governing the fluid flow in (r, θ, z) coordinate system (schlichting, 1968) are written as follows

$$\frac{\partial p}{\partial r} = 0, \tag{3.1}$$

$$\frac{\partial p}{\partial \theta} = 0, \tag{3.2}$$

$$\frac{\mu}{r}.\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{\partial p}{\partial z} \tag{3.3}$$

Where $u = u(r)$ denotes the axial velocity, μ is the viscosity of blood and p the pressure.

As a result of equation (3-5), the governing equation of fluid flow is given by

$$C + \frac{\mu}{r}.\frac{\partial}{\partial r}\left(r\frac{\partial \mu}{\partial r}\right) = 0 , \qquad kR_0 \le r \le R(z) \tag{3.4}$$

where $C = -\frac{dp}{dz}$, is the pressure gradient.

In order to study the annular flow of blood between the stenotic wall and a catheterized tapered artery, we shall consider the following cases:

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 9, Issue 1, January 2020

2. Boundary Conditions:

The boundary conditions for the present problem are given by

$$u(r) = u_s \text{ at r=R(z)}$$
$$u(r) = 0 \text{ at r=kR}_0 \quad (3.1)$$

Where u_s is the axial velocity slip at the stenotic wall (Biswas,2000)

3. Solutions of the problem:

The general integral of equation (3.4) is obtained as

$$\text{u(r)}+\frac{cr^2}{4\mu} = B + A\ln r, \qquad kR_0 \le r \le R(z) \quad (4.1)$$

Where A and B are constants of integrarion. As a result of applying condition (3.1) in equation (4.1) the expression for velocity function becomes

$$\text{u(r)} = \frac{C}{4\mu}\left[(kR_0)^2 - r^2\right] + \frac{\ln\left(\frac{r}{kR_0}\right)}{\ln\left(\frac{R(z)}{kR_0}\right)}\left[u_s + \frac{c}{4\mu}\left\{R(z)^2 - (kR_0)^2\right\}\right], kR_0 \le r \le R(z) \quad (4.2)$$

The rate of flow for the steady, incompressible and laminar flow is obtained by integrating the quantity

$$\text{Q} = 2\pi \int_{r=kR_0}^{R(z)} ru(r)dr \quad \text{which becomes with the help of the equation} \quad (4.2)$$

$$Q = \pi u_s\left[(R(z))^2 - \frac{(R(z))^2 - (kR_0)^2}{2\ln\left(\frac{R(z)}{kR_0}\right)}\right] + \frac{\pi c}{8\mu}\left[(R(z))^4 - (kR_0)^2 - \frac{\left\{(R(z))^2 - (kR_0)^2\right\}}{\ln\left(\frac{R(z)}{kR_0}\right)}\right] \quad (4.3)$$

Expression for wall stress is defined by

$$\tau_{R(z)} = -\mu\frac{\partial u(r)}{\partial r}]_{r=R(z)} \quad (4.4)$$

Which becomes with the help of equation (4.2)

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 9, Issue 1, January 2020

$$\tau_{R(z)} = \frac{cR(z)}{2} - \frac{\frac{\mu}{R(z)}}{\ln\left(\frac{R(z)}{kR_0}\right)}\left[u_s + \frac{c}{4\mu}\left\{\left(R(z)\right)^2 - \left(kR_0\right)^2\right\}\right] \qquad (4.5)$$

Apparent viscosity can be computed with the help of formula

$$\mu_a = \frac{\pi c\left(R(z)\right)^4}{8Q}$$

Where Q has the representation in equation (4.3)
Thus with the help of equation (4.3), we get

$$\mu_a = \left[\frac{8u_s}{c\left(R(z)\right)^2}\left[1 - \frac{1-\left(\frac{kR_0}{R(z)}\right)}{2\ln\left(\frac{R(z)}{kR_0}\right)}\right] + \frac{1}{\mu}\left\{-\left(\frac{kR_0}{R(z)}\right)^4 - \frac{\left\{1-\left(\frac{kR_0}{R(z)}\right)^2\right\}^2}{\ln\left(\frac{R(z)}{kR_0}\right)}\right\}\right] \qquad (4.6)$$

The non dimensional form of the flow variables and flow geometry can be expressed by using the following non-dimensional variables:

$$\bar{R} = \frac{R(z)}{R_0},\quad \bar{\delta_s} = \frac{\delta_s}{R_0}, u_0 = \frac{cR_0^{\,2}}{4\mu},\quad Q_0 = \frac{\pi c R_0^{\,4}}{8\mu}$$

$$\bar{Q} = \frac{Q}{Q_0}, \bar{u}_s = \frac{u_s}{u_0}, \bar{r} = \frac{r}{R_0}, \overline{kR_0} = \frac{kR}{R_0}, \bar{\tau}_{R(z)} = \frac{\tau_{R(z)}}{\tau_0}, \bar{\mu}_a = \frac{\mu_a}{\mu}$$

Velocity function:

$$\bar{u}(r) = \left[\left(\overline{kR_0}\right)^2 - \bar{r}^2\right] + \frac{\ln\left(\frac{\bar{r}}{\overline{kR_0}}\right)}{\ln\left(\frac{\bar{R}(z)}{\overline{kR_0}}\right)}\left[\bar{u}_s + \left\{\left(\bar{R}(z)\right)^2 - \left(\overline{kR_0}\right)^2\right\}\right], \overline{kR_0} \le \bar{r} \le \bar{R}(z), (4.7)$$

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 9, Issue 1, January 2020

Flow Rate:

$$\overline{Q}=\overline{u_s}\left[2\left(\overline{R}(z)\right)^2-\frac{\overline{R}(z)^2-\left(\overline{kR_0}\right)^2}{2\ln\left(\frac{\overline{R}(z)}{\overline{kR_0}}\right)}\right]+\left[\left(\overline{R}(z)\right)^4-\left(\overline{kR_0}\right)^4-\frac{\left\{\left(\overline{R}(z)\right)^2-\left(\overline{kR_0}\right)^2\right\}}{\ln\left(\frac{\overline{R}(z)}{\overline{kR_0}}\right)}\right],(4.8)$$

Wall shear stress:

$$\overline{\tau}_{R(z)}=\overline{R}(z)-\frac{1}{2\overline{R}(z)\ln\left(\frac{\overline{R}(z)}{\overline{kR_0}}\right)}\left[\overline{u_s}+\left\{\left(\overline{R}(z)\right)^2-\left(\overline{kR_0}\right)^2\right\}\right],(2.9)$$

Apparent viscosity:

$$\overline{\mu_a}=\left[\frac{2\overline{u_s}}{\left(\overline{R}(z)\right)^2}\left\{1-\frac{1-\left(\frac{\overline{kR_0}}{\overline{R}(z)}\right)}{2\ln\left(\frac{\overline{R}(z)}{kR_0}\right)}\right\}+\left\{1-\left(\frac{\overline{kR_0}}{\overline{R}(z)}\right)^4-\frac{\left\{1-\left(\frac{\overline{kR_0}}{\overline{R}(z)}\right)^2\right\}^2}{\ln\left(\frac{\overline{R}(z)}{\overline{kR_0}}\right)}\right\}\right]^{-1},(2.10)$$

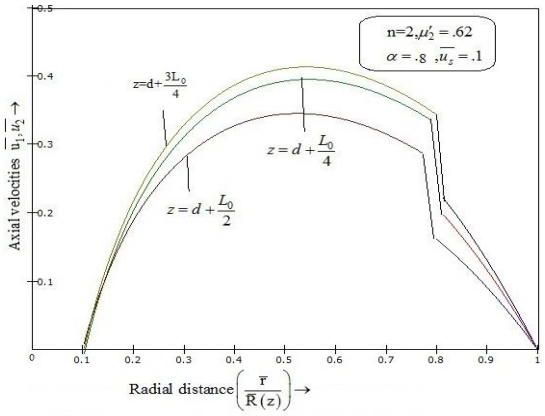

Fig.3 Variation of axial velocities against radial distance for different heights of stenosis $\overline{\delta_s}, \overline{\delta_l}$ and $\overline{u_s}=.1$, $KR_0=0.1$, n=2

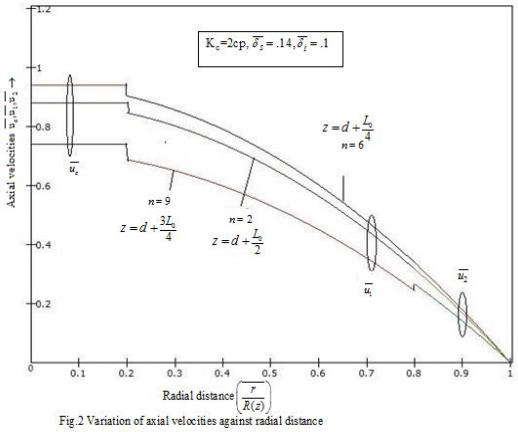

Fig.2 Variation of axial velocities against radial distance for different values of n and $\overline{u_s}=0.00$, $K_c=2cp$

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 9, Issue 1, January 2020

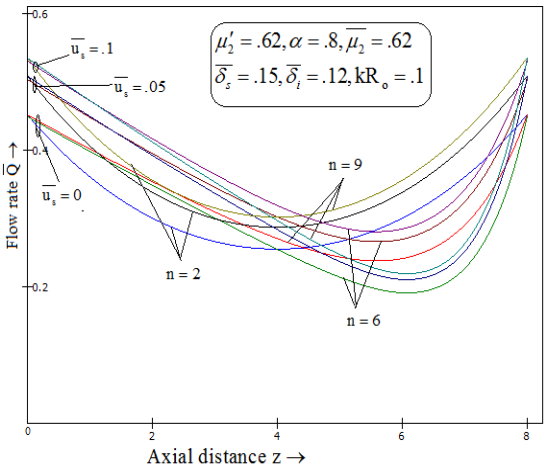

Fig 4:-Variation of flow rate against axial distance for n =2,6,9 ,and $\overline{u_s} = 0, .05, .1$

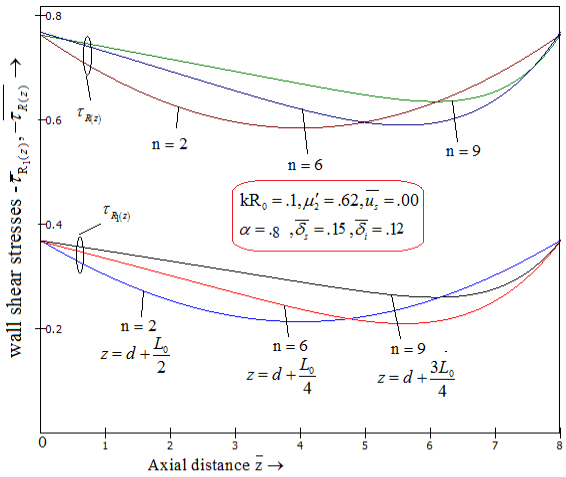

Fig 5 Variation of shear stress against axial distance for different values of n and $\overline{\delta_s} = .15, \overline{\delta_i} = .12$

III. CONCLUSION

In the present paper, steady flow of blood (a Newtonian fluid) through a catheterized stenosed artery subject to the condition of slip at the interface of fluids layers with mild asymmetric stenosis, velocity slip and zero-slip at the catheter and at the tube wall, has been investigated. Here three gradual advances of an abnormal growth (symmetric n = 2, n $\geq$ 2) and lumen of an artery cases of a slip and no-slip at interface and a catheter boundary are dealt with.

It may be worth mentioning that by employing a velocity slip at interface, wall shear stress may be reduced to a considerable extent that in term will help in reducing damage or rupture to the arterial endothelium. Also the reduction in wall shear stress enhancement in velocity and flow rate as a result of introducing at the axial velocity at the slip at interface, may be exploited for better functioning of the diseased arterial system and pressure flow-relationship in a uniform stenosed artery. Hence one may look forward for such device (drugs or tools) which could produce slip and used them for treatment and cure of PPL and arterial diseases as well as for rupture or damage to the arterial endothelium. Further the existing experimental work on blood flow through stenosed vessels, consider only pressure drop. It could be a matter of interest and important to determine the wall shear stress slip at interface in two-layered flow, velocity and flow rate etc. in the stenosed flow as well in the annular flow in a catheterized stenosed artery. Such investigation may be useful in determining the growth, development and progression of an arterial stenosis and investigating the pressure-flow relations and the behaviour of flow variables in the two-layered annular region caused by the invention of a catheter in an arterial stenosis and that in turn may be useful for better understanding of stenotic and arterial diseases like angina, pectoris, myocardial infarction, stroke, thrombosis and Hbss etc.

REFERENCES

1. Biswas, D.and Chakraborty, U.S.(2010).A Brief Review on Blood Flow Modeling in Arteries, Assam University Journal of Science &Technology,Silchar,India.Vol.6,pp.10-15
2. Biswas, D. (2000).Blood Flow Models: A comparative study , Mittal Publications, New Delhi, India.
3. Biswas, D.; Chakraborty, U.S. (2009). Steady Flow of Blood through a Catheterized Tapered Artery with stenosis. A Theoretical Model .Assam University Journal of Science and Technology. 4(2): 7-16
4. Biswas, D. and Paul, M. (2013) Study of Blood flow inside an Inclined non-uniform stenosed artery, International Journal of Mathematical Archive-4(5), 1-10

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 9, Issue 1, January 2020

5. Chaturani, P. and Biswas, D (1983), Effects of slip in Flow Through stenosed tube, Physiological Fluid Dynamics: Proc. Of 1st international Conf. on Physiological Fluid Dynamics ,September 5-7,pp. 75-80,IIT-Madras
6. Chakraborty, U.S., Biswas, D and Paul, M (2011) Suspension model blood flow through an inclined tube with an axially non-symmetrical stenosis, Korea- Australia Rheology, Vol-23, no. 1, pp. 25-32.
7. Chaturani, P. and D. Biswas, (1983).A theoretical study of blood flow through stenosed arteries with velocity slip at the wall. Proc. First Internat. Symposium on Physiological fluid Dynamics,IIT Madras, India, pp: 23-26
8. Schlichting, H., (1968) Boundary Layer Theory, Mc Graw-Hill Book Company, New York
9. Fung, Y.C. (1981) Biomechanics: Mechanical properties of Living Tissues, Springer-Verlag, New York Inc.
10. Mac Donald, D.A. (1979). On steady flow through modeled vascular stenosis. J. Biomech. 12:13-20
11. Mac Donald, D.A. (1974). Blood Flow in Arteries, Edward Arnold (Second Edition) London
12. Philip, D. and Chandra, P. (1996). Flow of eringen fluid (simple micro fluid) through an artery with mild stenosis. Int. J. Eng. Sci., 34: 87-99
13. Sankar, D.S.and Ismail, A.I. Md. (2009) Two-fluid mathematical models for blood flow in stenosed artery: A Comperative study. Boundary Value Problems 2009, 1-15
14. Mekheimer, Kh. S., El Kot, M. A, "Suspension model for blood flow through arterial catheterization", Chemical Engineering Cmmunications,197:9, 1195-1214, 2010. DOI: 10.10800098644090357 4883
15. Molls M, "Hyperthermia-the actual role in radiation oncology and future prospects", part I. Strahlen Onkol 168: 183-190, 1992
16. Sarma M. and Gaur R. K.," Modelling of MHD blood flow in a ballon Catheterized arterial stenosis with thermal radiation" , International Journal of Engineering science & research Technology, DOI: 10.5281/zenodo.1012463, 2017